Rapid Communications

Controlled parity switch of persistent currents in quantum ladders

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We investigate the behavior of persistent currents for a fixed number of noninteracting fermions in a periodic quantum ladder threaded by Aharonov-Bohm and transverse magnetic fluxes Φ and χ. We show that the coupling between ladder legs provides a way to effectively change the ground-state fermion-number parity, by varying χ. Specifically, we demonstrate that varying χ by 2π (one flux quantum) leads to an apparent fermion-number parity switch. We find that persistent currents exhibit a robust 4π periodicity as a function of χ, despite the fact that $\chi \to \chi + 2\pi$ leads to modifications of order $1/N$ of the energy spectrum, where N is the number of sites in each ladder leg. We show that these parity-switch and 4π periodicity effects are robust with respect to temperature and disorder, and outline potential physical realizations using cold atomic gases and photonic lattices, for bosonic analogs of the effects.

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Persistent currents [1,2] are among the most distinctive mesoscopic phenomena due to magnetic fields [3]. They represent a macroscopic manifestation of the electron phase coherence in metallic rings, and of the nonlocal effect of magnetic fields in quantum physics: When an Aharonov-Bohm (AB) flux Φ threads a ring, electrons acquire a phase Φ upon looping around it, responding with a current that persists at thermal equilibrium.

Recently, the effects of magnetic fields in quantum systems have been studied in numerous experiments focusing on bulk topological properties of fermions or bosons in two-dimensional (2D) lattices. With ultracold atoms [4], 2D fermionic [5–7] and bosonic [8–10] lattices under large transverse fluxes have been realized via synthetic gauge fields [11,12]. Analogs have been implemented in photonic lattices [13–17], with recent realizations of magnetic plaquettes hosting interacting photons in circuit cavity electrodynamics (circuit QED) [18,19].

Such experiments provide promising settings to investigate the *combined* effects of AB and transverse fluxes [20] in the mesoscopic realm. The minimal setup hybridizing 1D rings with AB flux and 2D lattices with a transverse field is a two-leg periodic quantum ladder threaded by both types of fluxes; see Fig. 1. Extensions to multileg ladders parallel the celebrated Laughlin's thought experiment for quantum Hall systems in cylinder geometry [20,21]. In the mesoscopic setting, intriguing questions arise such as how transverse fluxes affect persistent currents.

In this Rapid Communication, we show that the combination of AB and transverse magnetic fluxes in periodic two-leg ladders with fixed fermion number enables a controlled "parity switch" of persistent currents. We demonstrate that changing the transverse flux χ by a single quantum ($\chi \to \chi \pm 2\pi$) modifies persistent currents induced by the AB flux Φ as if the fermion-number parity was switched. We find that persistent currents become 4π instead of 2π periodic, as a function of χ, due to the coupling between ladder legs. We identify the conditions required for such phenomena, and discuss experimental platforms to observe them. The parity switch and 4π periodicity identified here are mesoscopic effects appearing in conventional quasi-1D quantum Hall systems [22–24], in particular, as we demonstrate by extensions to multileg ladders.

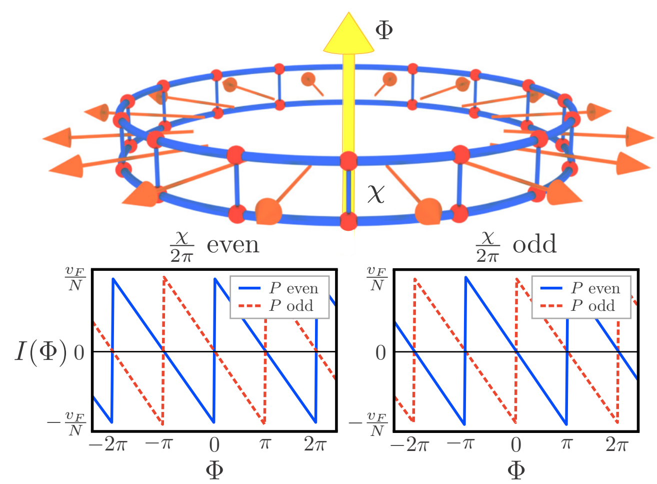

FIG. 1. Top: One-dimensional periodic quantum ladder threaded by Aharonov-Bohm flux Φ and transverse flux χ. Spinless fermion "hop" between lattice sites along and across ladder legs. This setup can be realized, e.g., using cold atomic gases or photonic lattices, for bosonic analogs. Bottom: Parity switch of persistent currents. When the parity of $\chi/(2\pi)$ is switched, persistent currents $I(\Phi)$ induced by Φ behave as if the fermion-number parity P was switched [$I(\Phi)$ is in units of the Fermi velocity v_F over the number N of sites per leg].

We consider the ladder system in Fig. 1, consisting of two tunnel-coupled rings of N sites: the "upper" (+) and "lower" (−) ladder legs, where each site hosts a spinless fermion. Without disorder and magnetic fluxes, each leg is described by the same quadratic Hamiltonian,

$$H_\sigma = \sum_{i,j} c^\dagger_{i,\sigma} \left(h_\parallel\right)_{ij} c_{j,\sigma}, \tag{1}$$

where $c_{i,\sigma}$ annihilates a fermion on site i of leg σ (with $i = 0,\dots,N-1$ and $\sigma = \pm$), and $h_\parallel$ is a σ-independent matrix. The coupling between ladder legs is described by

$$H_{+-} = \sum_{i,j} c^\dagger_{i,+}(h_\perp)_{ij}c_{j,-} + \text{H.c.} \tag{2}$$

The ladder Hamiltonian $H = H_+ + H_- + H_{+-}$ is number conserving, and is invariant under translations $i \to i+1$ along the ladder (by a lattice constant $a = 1$), and under time reversal ($h_\parallel$, $h_\perp$ are real matrices).

To induce and control persistent currents, we introduce two types of (real or synthetic) magnetic fluxes: (i) an AB flux Φ threading the entire ladder "loop," and (ii) a transverse flux χ threading the area between ladder legs; see Fig. 1. The flux Φ breaks time-reversal symmetry, inducing persistent currents $I(\Phi) = -\langle\partial_\Phi H\rangle/(2\pi)$ along the ladder [25], where $\langle\cdots\rangle$ denotes the expectation value over the relevant system state (the ground state, at zero temperature). In position space, Φ can be described as twisted boundary conditions $c_{N,\sigma} = e^{i\Phi}c_{0,\sigma}$, shifting the crystal momentum of the ladder globally ($k \to k + \Phi/N$). As opposed to Φ, the transverse flux χ has no analog in individual or decoupled chains: It induces no net current around the ladder but provides, as shown below, a key level of control over persistent currents induced by Φ. A natural gauge for χ is the Landau gauge [26], corresponding to a phase $e^{i\chi/N}$ for "hopping" from site i to $i+1$ along the upper ladder leg, or a global momentum shift $k \to k + \chi/N$ in the upper leg.

To maintain a degree of symmetry between ladder legs, we perform the gauge transformation $\tilde{c}_{j,\sigma} = e^{ij(\chi/2)/N}c_{j,\sigma}$, transferring half of the hopping phase $e^{i\chi/N}$ onto the lower leg while imposing modified twisted boundary conditions $\tilde{c}_{N,\sigma} = e^{i(\Phi+\chi/2)}\tilde{c}_{0,\sigma}$. In this "symmetric gauge," the flux Φ threads "symmetrically" the two "rings" corresponding to individual ladder legs, while χ threads both rings "antisymmetrically"—shifting momenta $k \to k + \sigma(\chi/2)/N$ with opposite signs in opposite legs. In momentum space, the flux-dependent ladder Hamiltonian takes the 2×2 matrix form

$$H(k,\chi) = \tilde{\mathbf{c}}^\dagger_k \begin{pmatrix} h_\parallel\left(k + \frac{\chi}{2N}\right) & h_\perp(k) \\ h^*_\perp(k) & h_\parallel\left(k - \frac{\chi}{2N}\right)\end{pmatrix}\tilde{\mathbf{c}}_k, \tag{3}$$

where the vector $\tilde{\mathbf{c}}_k \equiv (\tilde{c}_{k,+},\tilde{c}_{k,-})$ collects the momentum analogs of the operators $\tilde{c}_{j,\pm}$. Twisted boundary conditions lead to the quantization condition $k \in \{k_n = 2\pi n/N + \Phi/N + (\chi/2)/N\}$, where $n = 0,\dots,N-1$. Moreover, χ is constrained to multiples of 2π to ensure that (i) the Hamiltonian is periodic, and (ii) χ induces no current along the legs of the ladder when the interleg coupling is set to zero, as physically expected [27]. Shifts $\Phi \to \Phi + 2\pi m$ with integer m trivially shift the Brillouin zone, leaving the system invariant. Although shifts $\chi \to \chi + 4\pi m$ leave the set of allowed momenta invariant, shifting χ by 4π leads to modifications of the Hamiltonian of order $1/N$ [see Eq. (3)]. We thus expect physical properties to be invariant under shifts $\chi \to \chi + 4\pi$, up to corrections $\sim 1/N$.

Although fluxes break time-reversal (TR) symmetry, the Hamiltonian matrix in Eq. (3) is invariant under the effective TR symmetry defined by the antiunitary operator $\Theta = \sigma_x\mathcal{K}$, where $\mathcal{K}$ denotes complex conjugation, and σ_x is the standard

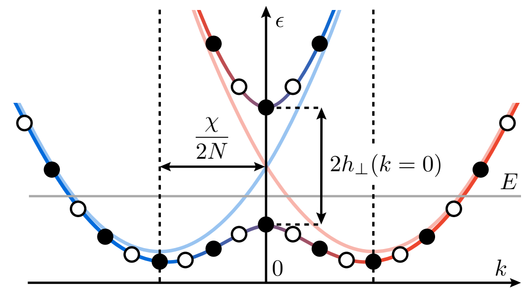

FIG. 2. Energy spectrum of a two-leg ladder around $k = 0$, at integer $\Phi/(2\pi)$. The transverse flux χ has two effects: (i) It shifts the bands corresponding to upper and lower ladder legs (red and blue faint lines) by χ/N with respect to each other; the interleg coupling $h_\perp(k)$ [Eq. (3)] then open "gaps" at the time-reversal-invariant momenta $k = 0$ and π (not shown) where bands cross, leading to hybridized bands (mixed red and blue lines). (ii) The flux χ controls allowed momenta (see text). Crucially, single-particle eigenstates are present at $k = 0$ when $\chi/(2\pi)$ is even [solid dots, open dots show allowed states at odd $\chi/(2\pi)$]. Therefore, the parity of $\chi/(2\pi)$ controls the state-number parity below an arbitrary energy level E (gray horizontal line) in the gap.

Pauli matrix. Since $\Theta^2 = +1$, states $|\psi(k)\rangle$ and $\Theta|\psi(k)\rangle$ with the same energy but opposite momenta need not belong to distinct bands (no Kramers' theorem). In particular, states at the TR-invariant (TRI) momenta $k = 0,\pi$ need not be degenerate, with crucial consequences detailed below. The system including twisted boundary conditions is invariant under Θ when $\Phi = \pi m$ and $\chi = 2\pi m$ (m integer).

When ladder legs are decoupled [$h_\perp(k) = 0$], Eq. (3) describes two bands $h_\parallel(k)$ shifted by $\chi/(2N)$ in opposite, σ-dependent k directions; see Fig. 2. If σ was a spin, χ would be a Rashba spin-orbit coupling [28–31]. Shifted bands cross at the TRI points $k = 0,\pi$ where states are crucially present or not depending on the *parity* of $\chi/(2\pi)$ [as $k_n = 2\pi n/N + (\chi/2)/N$, for integer $\Phi/(2\pi)$]. The coupling $h_\perp(k)$ opens "gaps" at the TRI momenta as a Zeeman field, lifting degeneracies; see Fig. 2. For integer $\Phi/(2\pi)$ and $\chi/(2\pi)$, the spectrum is twofold degenerate (with states at k and $-k$ related by Θ), *except* at the TRI points $k = 0,\pi$ where states are unique; see Fig. 3.

We now examine the effects of the transverse flux χ on persistent currents $I(\Phi)$ induced by small variations of Φ. Spectral degeneracies dictate the main features of $I(\Phi)$: They lead to a known "parity effect" where $I(\Phi)$ is either discontinuous or zero depending on the fermion-number parity [32–35]. The usual parity effect appears at $\chi = 0$ when the Fermi energy (energy of the highest occupied level) crosses the lowest band two times; see Fig. 3. The persistent current then reads $I(\Phi) = -(1/N)\sum_n v_{k_n}n_{k_n}$, where $v_k = \partial_k\varepsilon_k$ is the fermion velocity and n_k is the occupation distribution of single-particle eigenstates with energy ε_k. At $\Phi = 2\pi m$ (integer m), a pair of degenerate states lies at the Fermi energy. For odd fermion numbers, both states are occupied and the ground state is unique. Occupied states contribute with opposite fermion velocities, and $I(\Phi) = 0$. For even fermion numbers, instead, a single fermion is shared between degenerate states. The ground state is degenerate, and this degeneracy is lifted as soon as

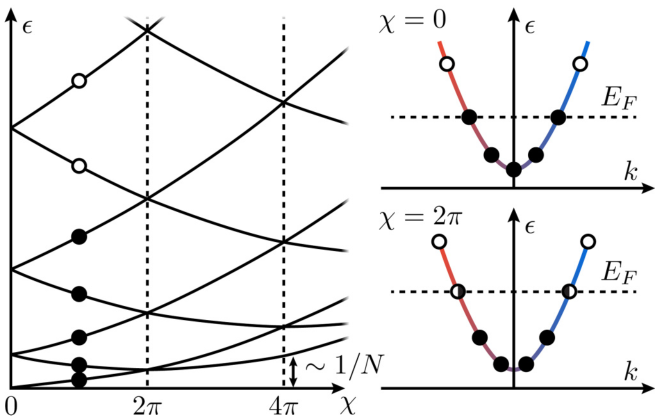

FIG. 3. Left: Single-particle spectrum (lower band in Fig. 2 for large interleg coupling) as a function of the transverse flux χ. The effective time-reversal symmetry Θ present at integer $\chi/(2\pi)$ (see text) imposes a twofold degeneracy of the entire spectrum, except at time-reversal-invariant momenta $k = 0,\pi$. Occupied/empty levels correspond to solid/open dots. Right: Single-particle eigenstates shown as a function of quasimomentum k, for $\chi = 0$ and 2π. When $\chi/(2\pi)$ alternates between even and odd parities, degenerate states at the Fermi energy E_F alternate between double and single occupation. Half-solid dots indicate states sharing a single particle. The spectrum is 4π periodic in χ, up to corrections $\sim 1/N$.

$\Phi \neq 2\pi m$, leading to a discontinuity in $I(\Phi)$. This behavior "switches" (with "odd"↔"even" above) when $\Phi \to \Phi \pm \pi$, leading to the "sawtooth" behavior in the lower left panel of Fig. 1. For $\Phi \in (-\pi,\pi]$, one finds

$$I(\Phi)_{\text{odd}} = -\frac{v_F}{N}\frac{\Phi}{\pi}, \quad I(\Phi)_{\text{even}} = \frac{v_F}{N}\text{sgn}(\Phi)\left(1 - \frac{|\Phi|}{\pi}\right). \tag{4}$$

We now show that inserting a *full* quantum of transverse flux χ leads to a similar "parity switch" as introducing *half* a quantum of AB flux Φ. This can be understood as follows: Figure 3 shows that nondegenerate states at TRI momenta become twofold degenerate when the parity of $\chi/(2\pi)$ is modified. Therefore, when a TRI state lies below the Fermi energy (typically the $k = 0$ state), higher-lying, twofold degenerate states at the Fermi energy must switch between single and double occupation when the parity of $\chi/(2\pi)$ changes, irrespective of the (fixed) fermion number. Up to modifications $\sim 1/N$ of the spectrum [of v_F in Eq. (4)], everything happens as if the fermion number was changed by ± 1, i.e., as if the fermion-number parity was switched. This parity switch is illustrated in Fig. 3: When $\chi/(2\pi)$ is even, the TRI momentum $k = 0$ is allowed, the number of states below E_F is odd, and the usual behavior of persistent currents [Eq. (4)] is observed. When χ is odd, instead, $k = 0$ is forbidden and the number of states below E_F is even. The behavior of $I(\Phi)$ then changes as if the fermion-number parity had been switched (lower right panel of Fig. 1). A quantitative analysis of this effect in a minimal lattice model [7,36–38] can be found in the Supplemental Material [39].

The above results imply that persistent currents exhibit a remarkable 4π periodicity in χ: Although changes $\chi \to \chi + 4\pi$ lead to $O(1/N)$ modifications of the bands, the main features of $I(\Phi)$—its zeros and discontinuities—are *strictly* 4π periodic in χ. These features are dictated by energy crossings between single-particle eigenstates, which are strictly enforced by the symmetry Θ at integer values of $\chi/(2\pi)$. Figure 4 shows the current $I(\Phi)$ as a function of Φ and χ, for a minimal lattice model [39]. As expected, $I(\Phi)$ features a robust 4π periodicity in χ.

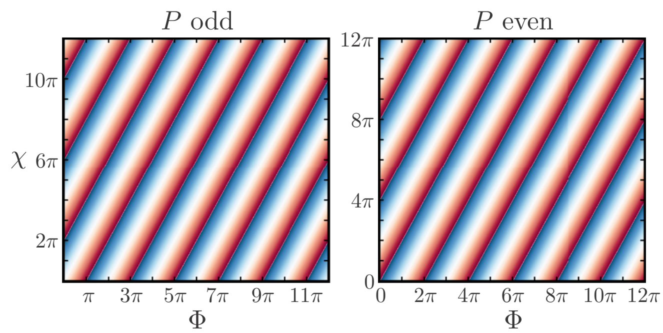

FIG. 4. Density plots of the persistent current as a function of the AB flux Φ and the transverse flux χ, for even/odd fermion-number parity P (left/right). The fermion number is fixed with Fermi energy in the "gap" at $k = 0$ (as the level E in Fig. 2). Positive/negative values are shown in blue/red. Plots correspond to a minimal lattice model with unit hopping strength between nearest-neighboring sites along and across ladder legs [39]. The left (right) plot corresponds to 61 (60) particles with system size $N = 150$. Changing P causes a relative shift $\chi \to \chi \pm 2\pi$ and small differences $\sim 1/N$ that are not visible here.

Next, we comment on the robustness of the parity-switch and 4π periodicity effects with respect to temperature, disorder, and system size. We recall that these effects stem from the effective TR symmetry Θ present at integer values of $\chi/(2\pi)$. Since temperature does not affect this symmetry, it does not lift the effects, but affects their visibility: A temperature $T > 0$ spreads the occupation distribution of single-particle eigenstates, thereby decreasing the overall amplitude of $I(\Phi)$; the patterns in Fig. 4 remain visible as long as T is lower than the typical energy separation $\sim E_F/N$ between states at the Fermi energy. Disorder has similar effects if it preserves the Θ symmetry. If it breaks it, instead, discontinuities of $I(\Phi)$ are smoothed out [34,40], and parity switches need not occur at integer values of $\chi/(2\pi)$ anymore, leading to an approximate (on average) 4π periodicity of $I(\Phi)$ in χ. Details are provided in the Supplemental Material [39].

Our results apply to ladders with $L > 2$ legs where topological effects provide enhanced robustness against disorder. For $h_\perp = 0$, each leg contributes a single band to the ladder spectrum: Each band has a well-defined leg index $l = 0,\ldots,L-1$, and neighboring bands are shifted by $(\chi/N)/(L-1)$ with respect to each other, as shown in Fig. 2 for $L = 2$. The interleg coupling $h_\perp$ mixes bands and opens gaps decreasing exponentially with $|l_1 - l_2|$ at crossings between bands l_1 and l_2 [39]. One can distinguish two pictures depending on the value of χ:

(i) For $\chi/N \lesssim 2\pi$, the L minima of the L momentum-shifted bands all "fit" within the first Brillouin zone, and band crossings at the nth lowest energy occur between bands with index difference $|\Delta l| = n$ (for small n). The size of the lowest-energy gap ($n = 1$) is of order $|h_\perp|$, while higher-energy gaps are exponentially smaller [39]. Hybridized bands can be seen as "Landau levels," as in the coupled-wire construction for quantum Hall systems introduced in Ref. [24]. As we show in the Supplemental Material [39], parity switches induced by integer changes of $\chi/(2\pi)$ are observed *whenever the number* ν *of occupied hybridized bands (or Landau levels) is an odd integer*. In that case, the multileg ladder exhibits an odd number ν of chiral edge modes on each side of its cylinder geometry (around extremal legs) [24]. The integer ν and the edge modes have a topological nature [26,41]: The overlap between counterpropagating modes on opposite edges becomes exponentially small with increasing L, which suppresses disorder effects.

(ii) When $\chi/N \gtrsim 2\pi$, bands corresponding to individual ladder legs can shift beyond the first Brillouin zone and "backfold" into it. When χ becomes of the order of one flux quantum per unit cell, the system enters the Harper-Hofstadter quantum Hall regime [22,23], where gaps are determined by the fraction p/q of flux quantum per unit cell (q being a prime integer, and p taking values between 1 and q). As in the low-flux case (i), topological chiral edge modes typically appear in these gaps, and robust parity switches are observed when the number ν of occupied modes per edge is odd. Disorder-induced scattering is similarly suppressed with increasing L [39].

Generalizations to multileg ladders show that the parity switch and 4π periodicity are *mesoscopic* analogs of Landau quantization effects [42], where additional quanta of transverse flux χ introduce additional states ("cyclotron orbitals") for free fermions in Landau levels. While fluxes of the order of one flux quantum per unit cell are required to observe such effects [43], the phenomena identified here are accessible at arbitrarily low flux χ. They crucially differ from the well-established generation of persistent currents via χ in Landau systems, through the nonvanishing Hall conductivity.

Cold atoms in optical lattices are prime candidates to realize periodic ladders with AB and transverse fluxes. An implementation of our setup (Fig. 1) was recently proposed in cylindrical lattices generated by Laguerre-Gauss beams (rotated to induce a synthetic flux Φ) [20]. Periodic ladders with synthetic fluxes Φ and χ can also be realized using internal degrees of freedom to simulate one or both spatial dimensions [6,44,45]. In particular, periodicity along synthetic dimensions can be obtained by coupling extremal internal states [46,47]. The effects studied in this work can be observed in short ladders with few lattice sites [39], in any system with Hamiltonians similar to Eq. (3), with effective TR symmetry Θ. The details of our simplified tight-binding model are irrelevant. The underlying lattice structure is not even required: Our results would similarly apply to coupled one-dimensional continuum rings realized, e.g., with bosonic superfluids [48–52] coupled to each other [53]. To observe the parity-switch and 4π periodicity effects in cold-atom experiments, one challenge is to suppress particle-number fluctuations between measurements: Measuring $I(\Phi)$ for different Φ and χ with parity fluctuations leads to a statistical average between patterns in Fig. 4 [54]. The average fermion-number parity determines the visibility of the effects, and vice versa. In particular, the periodicity of $I(\Phi)$ in χ is reduced from 4π to 2π when the average parity vanishes [39].

Periodic ladders with synthetic fluxes can also be realized with bosons in photonic lattices [17,55,56]. Though there is no Pauli blocking to suppress photon-number fluctuations, strong interactions may lead to analogs of persistent currents [19], and to effective chemical potentials [57]. Regardless, noninteracting photons described by the ladder Hamiltonian in Eq. (3) (with bosonic operators) exhibit the same approximate 4π periodicity in χ, leading to a similar periodicity of physical observables. Decoherence due to photon losses can be suppressed in short ladders as in Ref. [19], e.g., where currents were probed in a circuit-QED system [39].

We have shown that the insertion of transverse-flux quanta $\chi/(2\pi)$ in quantum systems with a ladder or, equivalently, Corbino disk geometry provides a robust way to perform controlled parity switches revealed by mesoscopic quantities such as persistent currents. Such effects are accompanied by a remarkable 4π periodicity of physical observables in χ, up to corrections of order $1/N$. These phenomena are robust to local interactions: As long as the Fermi wave vector k_F is not commensurate with the lattice, weak interactions renormalize the Fermi velocity $v_F \to u$ (see, e.g., Refs. [37,38]), and parity switches survive, with renormalized persistent currents in Eq. (4) [58].

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