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[9]For an equatorial circular orbit in a Kerr spacetime, $K = K^{N} + \chi_{\pm} K^{N}\eta$, where $\chi_{\pm} = (1-3\phi \pm 2\phi^{-1/2}\psi)^{-1} \times (1 \mp \phi^{-3/2}\psi)^2\phi$. Hence $\chi_{\pm} > 0$ for all circular orbits and diverges as the null orbit is approached. It is interesting to note that $\chi_{\pm} = \frac{1}{3}$ at the last stable circular orbit.

[10]See Ref. 4. Van Patten and Everitt have proposed an experiment to measure the effect of the dragging of inertial frames: R. A. Van Patten and C. W. F. Everitt, Phys. Rev. Lett. 36, 629 (1976), and Celest. Mech. 13, 429 (1976).

[11]As a reminder of this fact a bar is placed over indices referring to these tetrads. The nonzero components of $\Lambda_{(\bar{r})}{}^{\mu}$ and $\Lambda_{(\bar{\theta})}{}^{\mu}$ are given by $[\Lambda_{(\bar{r})}{}^{r} = \alpha$, $r\Lambda_{(\bar{\theta})}{}^{\theta} = \alpha\gamma$, $\alpha\Lambda_{(\bar{\theta})}{}^{t} = \gamma\phi^{1/2}$,

$$r\Lambda_{(\bar{r})}{}^{\varphi} = \alpha\phi^{-3/2}\psi[\cot(\omega\tau) - \csc(\omega\tau)\cos(\omega_0\tau)],$$

and

$$r\Lambda_{(\bar{\theta})}{}^{\varphi} = \alpha\phi^{-3/2}\psi[\csc(\omega\tau)\sin(\omega_0\tau) - \beta],$$

where $\beta = \gamma(\alpha^2 - \phi\alpha^{-2})$. The components of $\Lambda_{(\bar{\varphi})}{}^{\mu}$ may be determined from the orthonormality conditions. It is important to note that the rotation of the platform by frequency ω_0 only helps simplify the final result and is not the cause of the *main* effects considered in this paper.

[12]The angular momentum of the source has been taken into account only to first order: therefore, the linear results may break down for $\tau \gtrsim \Omega^{-1}$.

[13]To ensure this, a sufficiently drag-free laboratory system is necessary just as in the gyroscope experiment.

[14]It should be pointed out that our results differ from those of Ref. 7 in several important respects: (i) The Schwarzschild effect is not mentioned in Ref. 7, (ii) the amplitude of the rotation-dependent effect is not given correctly since the secular term is absent, and (iii) the opposite conclusion is implied regarding the question of measurability of the relativistic effects.

Influence of Dissipation on Quantum Coherence

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A quantum mechanical particle which moves in a symmetric double-well potential, and whose interaction with the environment is described in the classical regime by a phenomenological friction coefficient η, is considered. It is shown that, provided $\eta q_0^2/\hbar$ exceeds a critical value of order unity ($\pm q_0$ are the locations of the potential minima), the mean rate of tunneling between the degenerate minima decreases with temperature, leading at $T = 0$ to spontaneous symmetry breaking.

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There has been recent interest[1,2] in the influence of dissipation on quantum *tunneling* out of a metastable state. Here we consider the related problem of quantum *coherence.* Specifically we consider a particle of mass M moving in a symmetric double-well potential $V(q) = V(-q)$ which has minima at $q = \pm q_0$ and a local maximum at $q = 0$, and whose classical equation of motion is $M\ddot{q} + \eta\dot{q} = -dV/dq + F_{\text{ext}}(t)$. We limit our considerations to temperatures small compared to the frequency $\omega_0 = [M^{-1}V''(q_0)]^{1/2}$ of small oscillations around one of the minima: $k_B T \ll \hbar\omega_0$, the limit in which thermal activation over the barrier can be neglected compared with quantum tunneling. Further, in the limit $\hbar\omega_0 \ll \Delta V = V(0) - V(q)$, one can truncate to the lowest two states ψ_s, ψ_a with energies E_s, E_a, respectively. If the system is prepared at $T = 0$ and time $t = 0$ in the state $\psi_L = (\psi_a + \psi_s)/\sqrt{2}$ representing a wave packet localized in the left-hand well, the amplitude for being in the left-hand well at time t oscillates with frequency $\Delta_0/\hbar = (E_a - E_s)/\hbar$. This is the phenomenon of "quantum coherence." At finite temperatures this oscillation is reflected in the correlation function (with $\beta = 1/k_B T$)

$$\langle q(t)q(0)\rangle = q_0^2\{\cos(\Delta_0 t/\hbar) - i\sin(\Delta_0 t/\hbar)\tanh(\tfrac{1}{2}\beta\Delta_0)\}. \tag{1}$$

In this Letter we consider the effect of dissipation on quantum coherence. Our calculations are restricted to the regime where $\eta q_0^2/\hbar$ exceeds a critical value of order unity, while for computational simplicity we take η sufficiently small that the classical motion about one of the minima is lightly damped ($\eta \ll M\omega_0$), although the latter is not a fundamental limitation. The principal conclusions are as follows:

(i) At $T=0$ there is spontaneous symmetry breaking, and a twofold degenerate ground state, provided $\eta q_0^2/\hbar$ exceeds a critical value of order unity. As η increases there is a "phase transition" from a "disordered" ($\langle q\rangle = 0$) to an "ordered" ($\langle q\rangle \neq 0$) phase which is in the same universality class as the one-dimensional Ising model with inverse-square-law interactions.[3]

(ii) With decreasing temperature the tunneling between the two degenerate minima slows down in anticipation of the zero-temperature transition. On the basis of a simple approximate calculation combined with a renormalization-group (RG) treatment we predict that the mean tunneling rate is given approximately by

$$\Delta_{\rm eff}(T)/\hbar \simeq (\beta\Delta_0^2/4\eta q_0^2)(k_B T/\hbar\omega_0)^{\varphi}, \tag{2}$$

where $\varphi \simeq 4\eta q_0^2/\pi\hbar$ for $\eta q_0^2/\hbar \gg 1$, but depends on Δ_0 in general.

(iii) There is a complete loss of phase coherence.[4] The system continues to tunnel, but in a random (rather than "clocklike") manner.

Following Caldeira and Leggett,[1] the dissipative interaction of the system with the environment is modeled by a linear coupling to a set of harmonic oscillators. The Hamiltonian is

$$H = \tfrac{1}{2}M\dot{q}^2 + V(q) + \tfrac{1}{2}\sum_\alpha m_\alpha \dot{x}_\alpha^2 + \tfrac{1}{2}\sum_\alpha m_\alpha\omega_\alpha^2 x_\alpha^2 + q\sum_\alpha c_\alpha x_\alpha + q^2\sum_\alpha c_\alpha^2/2m_\alpha\omega_\alpha^2. \tag{3}$$

The final term in Eq. (3) has been included so that there is no shift in the bare potential $V(q)$ due to the coupling, which merely serves (if the spectrum of oscillator frequencies is appropriately chosen) to introduce a dissipative term in the classical equation of motion for q. This point has been the subject of some recent discussion.[5] The partition function can be obtained by the Feynman path-integral method[6] as a functional integral over functions $q(s)$, $\{x_\alpha(s)\}$ defined in *imaginary time,* $0 \leq s < \beta$,

$$Z = \int Dq(s)\int \pi_\alpha Dx_\alpha(s)\exp\left\{-\int_0^\beta ds\left[\frac{M}{2\hbar^2}\left(\frac{dq}{ds}\right)^2 + V(q) + \sum_\alpha \frac{m_\alpha}{2\hbar^2}\left(\frac{dx_\alpha}{ds}\right)^2 + \tfrac{1}{2}\sum_\alpha m_\alpha\omega_\alpha^2 x_\alpha^2 + q\sum_\alpha c_\alpha x_\alpha + q^2\sum_\alpha \frac{c_\alpha^2}{2m_\alpha\omega_\alpha^2}\right]\right\}, \tag{4}$$

with periodic boundary conditions $q(s+\beta) = q(s)$, $x_\alpha(s+\beta) = x_\alpha(s)$. The functions $q(s)$, $\{x_\alpha(s)\}$ may be expressed as Fourier series, $q(s) = \sum_{n=-\infty}^{\infty} q_n \exp(i\omega_n s)$, etc., where $\omega_n = 2\pi n/\beta$.

The oscillator degrees of freedom are integrated out to leave, up to a multiplicative constant the partition function of the oscillators),

$$Z = \int Dq(s)\exp\left\{-\int_0^\beta ds\left[\frac{M}{2\hbar^2}\left(\frac{dq}{ds}\right)^2 + V(q)\right] - \beta\sum_n \alpha(n) q_n q_{-n}\right\}, \tag{5}$$

where

$$\alpha(n) = (\omega_n/\hbar)^2\sum_\alpha \frac{c_\alpha^2}{2m_\alpha\omega_\alpha^2[(\omega_n/\hbar)^2 + \omega_\alpha^2]} = \frac{\omega_n^2}{\pi\hbar^2}\int_0^\infty \frac{d\omega J(\omega)\omega^{-1}}{(\omega_n/\hbar)^2 + \omega^2}.$$

For the classical motion to be determined by a well-defined friction coefficient η, we must have $J(\omega) = \eta\omega$, giving

$$\alpha(n) = \eta|\omega_n|/2\hbar. \tag{6}$$

Substituting (6) into (5), and retransforming the final term to imaginary time, gives the final result [equivalent to Eq. (10) of Ref. 1]

$$Z=\int Dq(s)\exp[-S_{\rm eff}\{q(s)\}],$$
$$S_{\rm eff}\{q(s)\}=\int_0^\beta ds\left[\frac{M}{2\hbar^2}\left(\frac{dq}{ds}\right)^2+V(q)\right]+\frac{\pi\eta}{4\hbar\beta^2}\int_0^\beta ds\int_0^\beta ds'\,\frac{[q(s)-q(s')]^2}{\sin^2[(\pi/\beta)(s-s')]}\,. \tag{7}$$

In the absence of dissipation, and at $T=0$, the tunneling frequency can be obtained by standard "instanton" methods,[7] via a calculation of the local minimum of the functional $S_{\rm eff}\{q(s)\}$ in function space. For $\eta=0$, an extremum of $S_{\rm eff}$ satisfies

$$(M/\hbar^2)d^2q/ds^2=dV/dq$$

which is a classical equation of motion corresponding to the inverted potential $\tilde{V}(q)=-V(q)$. For the nontrivial "kink" (or "instanton") solution the particle falls off its unstable potential maximum at $q=-q_0$ and reaches $q=+q_0$ as $s\to\infty$. For the corresponding "antikink" solution the particle starts at $q=+q_0$ and finishes at $q=-q_0$. If S_0 is the value of $S_{\rm eff}$ corresponding to a single kink (or antikink), the kink contribution to the partition function can be written as[7]

$$Z=\sum_{n=0}^\infty\int_0^\beta\frac{ds_{2n}}{\tau}\int_0^{s_{2n}}\frac{ds_{2n-1}}{\tau}\cdots\int_0^{s_2}\frac{ds_1}{\tau}\exp(-2nS_0)=\sum_{n=0}^\infty[(\beta/\tau)^{2n}/(2n)!]\exp(-2nS_0)=\cosh(\tfrac{1}{2}\beta\Delta_0), \tag{8}$$

where

$$\Delta_0=(2/\tau)\exp(-S_0). \tag{9}$$

In Eqs. (8) and (9), $\tau\sim(\hbar\omega_0)^{-1}$ is the kink "width" ($\beta/\tau\gg1$), and from (8) we deduce that Δ_0 is the energy splitting of the lowest two states so that the frequency of tunneling between the two minima of $V(q)$ is just $\Delta_0/\hbar$.

For the case $\eta=0$ discussed above, the kinks and antikinks form a noninteracting gas. The effect of dissipation is to introduce kink-kink interactions via the final term in Eq. (7). For a preliminary analysis we treat this term perturbatively. Working in Fourier space, using Eqs. (5) and (6), we obtain for the two-point correlation function, correct to $O(\eta)$,

$$\langle q_nq_{-n}\rangle=\langle q_nq_{-n}\rangle_0-(\beta\eta/2\hbar)\sum_m|\omega_m|\{\langle q_nq_{-n}q_mq_{-m}\rangle_0-\langle q_nq_{-n}\rangle_0\langle q_mq_{-m}\rangle_0\}, \tag{10}$$

where $\langle\ldots\rangle_0$ indicates a thermal average in the noninteracting ($\eta=0$) theory. Such averages are most readily carried out in imaginary time:

$$\langle q(s_1)q(s_2)\rangle_0=q_0^2\,{\rm sech}(\tfrac{1}{2}\beta\Delta_0)\cosh[\tfrac{1}{2}\Delta_0(\beta+2s_1-2s_2)],\quad s_1<s_2, \tag{10a}$$

$$\langle q(s_1)q(s_2)q(s_3)q(s_4)\rangle_0=q_0^4\,{\rm sech}(\tfrac{1}{2}\beta\Delta_0)\cosh[\tfrac{1}{2}\Delta_0(\beta+2s_1-2s_2+2s_3-2s_4)],\quad s_1<s_2<s_3<s_4. \tag{10b}$$

Note that (10a) is identical to (1) after the continuation $s_2-s_1\to it/\hbar$. The Fourier transforms required for Eq. (10) have been derived by Stinchcombe.[8]

Specialized to the limit $\beta\Delta_0\ll1$, the result can be written, correct to $O(\eta)$, as

$$\langle q_nq_{-n}\rangle=q_0^2\Delta^2[\Delta^2+\omega_n^2+(4\eta q_0^2/\beta\hbar)|\omega_n|]^{-1}, \tag{11}$$

where

$$\Delta=\Delta_0[1-(4\eta q_0^2/\beta\hbar)\sum_{m=1}^{m_c}\omega_m^{-1}], \tag{12}$$

and m_c is a cutoff determined from $2\pi m_c/\beta=\hbar\omega_c\sim\tau^{-1}\sim\hbar\omega_0$, i.e., $m_c\sim\beta\hbar\omega_0$, up to constants of order unity. The cutoff reflects the fact that $q(s)$ cannot change on a scale smaller than the kink width τ. Since $\beta\hbar\omega_0\gg1$, Eq. (11) gives

$$\Delta\simeq\Delta_0[1-(2\eta q_0^2/\pi\hbar)\ln(\beta\hbar\omega_0)]\simeq\Delta_0(k_BT/\hbar\omega_0)^{2\eta q_0^2/\pi\hbar}. \tag{13}$$

The exponentiation is merely suggestive at this stage, but will be placed on a sounder footing by the RG treatment which follows.

Although Eq. (11) is only exact to $O(\eta)$, the form in which it is written (which corresponds to the random-phase approximation for the equivalent spin-$\tfrac{1}{2}$ problem[8]) is extremely suggestive, since it is identical to the correlation function for a damped harmonic oscillator, with natural frequency Ω and decay constant γ, with the identifications

$$\Omega = \Delta/\hbar, \quad \gamma = 4\eta q_0^2/\beta\hbar^2.$$

Thus $\gamma/\Omega = 4\eta q_0^2/\beta\hbar\Delta \to \infty$ as $T \to 0$ provided $2\eta q_0^2/\pi\hbar > 1$. The "equivalent oscillator" is heavily damped, therefore, as $T \to 0$ even though the classical motion of the real particle may be lightly damped. Also $\hbar\Omega^2/\gamma = \beta\Delta^2/(4\eta q_0^2/\hbar) \ll k_B T$ as $T \to 0$ so that the "equivalent oscillator" may be treated classically in this regime, giving for the correlation function in *real time*

$$\langle q(t)q(0)\rangle \simeq q_0^2 \exp(-\beta\Delta^2 t/4\eta q_0^2). \tag{14}$$

The absence of an oscillating part in Eq. (14) implies a total loss of phase coherence due to dissipation. Indeed Eq. (14) is suggestive of a Poisson process corresponding to a mean tunneling rate

$$\Delta_{\rm eff}(T)/\hbar = \beta\Delta^2/4\eta q_0^2 \simeq (\beta\Delta_0^2/4\eta q_0^2)(k_B T/\hbar\omega_0)^{4\eta q_0^2/\pi\hbar}. \tag{15}$$

In this approximation the mean tunneling rate vanishes as $T \to 0$ since $2\eta q_0^2/\pi\hbar > 1$ by assumption.

These results can be placed on a somewhat firmer footing with an RG approach. The final term in Eq. (7) is first simplified. Replacing the sine by its argument and integrating twice by parts utilizing the periodicity of $q(s)$ yields

$$S_{\rm eff} = \int_0^\beta ds \left[\frac{M}{2\hbar^2}\left(\frac{dq}{ds}\right)^2 + V(q)\right] - \frac{\eta}{2\pi\hbar}\int_0^\beta ds \int_0^\beta ds' \left(\frac{dq}{ds}\right)\left(\frac{dq}{ds'}\right) \ln\left(\frac{|s-s'|}{\tau_0}\right), \tag{16}$$

with τ_0 arbitrary. The "kink approximation" to dq/ds is

$$dq/ds = 2q_0 \sum_i \epsilon_i \delta(s - s_i) \tag{17}$$

corresponding to kinks ($\epsilon_i = 1$) or antikinks ($\epsilon_i = -1$) at locations s_i. The delta functions in (17) strictly have a finite width τ equal to the kink width, $\tau \sim (\hbar\omega_0)^{-1}$. Substituting (17) into the final term of (16) yields, for the partition function,

$$Z = \sum_{n=0}^{\infty} \left(\frac{\tilde{\Delta}_0}{2}\right)^{2n} \int_0^\beta \frac{ds_{2n}}{\tau} \int_0^{s_{2n}-\tau} \frac{ds_{2n-1}}{\tau} \cdots \int_0^{s_2-\tau} \frac{ds_1}{\tau} \exp\left\{\varphi_0 \sum_{i>j} (-1)^{i-j} \ln\left(\frac{s_i - s_j}{\tau}\right)\right\}, \tag{18}$$

where

$$\tilde{\Delta}_0 = \Delta_0\tau, \quad \varphi_0 = 4\eta q_0^2/\pi\hbar.$$

Equation (18) is the generalization of Eq. (8) to the interacting case. The physical choice $\tau_0 = \tau$ has been made, and the logarithmic interaction cut off at $|s_i - s_j| = \tau$, since distinct kinks are not well defined if their separation is less than their width.

Equation (18) is the same starting point as that used for the RG treatments of the Kondo problem and the one-dimensional Ising model with inverse-square-law interactions.[9] RG recursion relations for $\tilde{\Delta}$ and φ may be obtained in the standard way as a function of the cutoff τ. Following Anderson and Yuval[9] we obtain

$$d\tilde{\Delta}/d\ln\tau = (1 - \varphi/2)\tilde{\Delta}, \tag{19}$$

$$d\varphi/d\ln\tau = -\varphi\tilde{\Delta}^2. \tag{20}$$

These equations are valid for $\tilde{\Delta} \ll 1$ which for $\tau \lesssim \beta$ requires $\beta\Delta_0 \ll 1$ as assumed earlier. The RG flows in the $(\tilde{\Delta}, \varphi)$ plane which result from these equations are well known.[9] There is a separatrix $\tilde{\Delta}^2 = \varphi - 2 - 2\ln(\varphi/2)$ which separates flows to a line of stable fixed points $\tilde{\Delta} = 0$, $\varphi \geq 2$ from flows which take the parameters outside the domain of validity of the equations (i.e., make $\tilde{\Delta}$ grow). We concentrate here on the case where the flow is to the line of stable fixed points, which occurs for $\varphi_0 > 2$ and sufficiently small $\tilde{\Delta}_0$ [i.e., $\tilde{\Delta}_0^2 < \varphi_2 - 2 - 2\ln(\varphi_0/2)$]. The simplest case is $\varphi_0 \gg 2$. Then φ may be treated as constant in Eq. (19). Integrating from $\tau = (\hbar\omega_0)^{-1}$ to $\tau = \beta$ gives the running value $\tilde{\Delta}(T)$ as

$$\tilde{\Delta}(T) = \tilde{\Delta}_0(k_B T/\hbar\omega_0)^{\varphi_0/2-1},$$

or

$$\Delta(T) = \Delta_0(k_B T/\hbar\omega_0)^{\varphi_0/2}, \tag{21}$$

in agreement with Eq. (13) which, although derived by expansion in η, is thus seen to be correct also for $2\eta q_0^2/\pi\hbar \gg 1$.

The vanishing of the mean tunneling rate as $T \to 0$ is the precursor of a spontaneous symmetry breaking at $T = 0$. The ground-state average of the position coordinate plays the same role as the spontaneous magnetization m of the corresponding one-dimensional Ising model with $1/r^2$ interactions. Taking over the result from the latter model,[3] we predict that as a function of η, $\langle q\rangle$ will at a critical value $\eta_c \sim h/4q_0^2$ change discontinuously from zero to a nonzero value. (This means that for $\eta > \eta_c$ the ground state is twofold degenerate.) Equivalently, we can say that a particle initially localized in, e.g., the left-hand well has a greater than 50% chance of being found in the same well after infinite time.

The most likely system for an experimental test of these ideas is the SQUID,[10] in which the flux through the ring plays the role of the coordinate q. For the case where the two minima differ by nearly a whole flux quantum, it seems likely that tunneling rates will be too small to observe, but by varying the system parameters it may be possible to arrange for the two minima to be separated by a small fraction of a flux quantum, making observation much more likely.

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Axions and Family Symmetry Breaking

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Possible advantages of replacing the Peccei-Quinn U(1) quasisymmetry by a group of genuine flavor symmetries are pointed out. Characteristic neutral Nambu-Goldstone bosons will arise, which might be observed in rare K or μ decays. The formulation of Lagrangians embodying these ideas is discussed schematically.

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In order to understand why the strong interaction does not exhibit large CP-invariance violations, it is desirable to postulate that conservation of axial baryon number[1] $\mathrm{U}(1)_A$ is violated spontaneously, except for (soft) instanton effects.[2-4] Phenomenological considerations then require that this symmetry be broken at a very large scale,[5] leading among other things to the emergence of exceedingly light, exceedingly weakly coupled particles ("invisible axions")[6] which are essentially the Nambu-Goldstone bosons associated with the quasisymmetry $\mathrm{U}(1)_A$.

Reflecting on this scheme, I think we can identify two unsatisfying features:

(i) Axial baryon number is only one small part of a very large flavor symmetry group that emerges when quark masses are neglected. Why should it be treated on such a special footing?

(ii) The requirement that the theory exhibit a symmetry "except for instanton effects" seems an artificial one. After all, instantons refer to a method of calculation and not to an intrinsic element of the theory. Although one can partially justify the separation of instanton effects on the basis that they are very soft (disappearing rapidly at high momentum scales),[4] it would seem more satisfactory to have the offensive interaction terms banished for real symmetry reasons.

These points reinforce one another, since enlargement of the $\mathrm{U}(1)_A$ symmetry might automatically forbid the dangerous terms which previously were banished by appeal to the $\mathrm{U}(1)_A$ quasisymmetry.

Actually, point (i) should be viewed in a more